MAC 2311
Integrals Worksheet (Spring 2020)
Kincade
(with Trig)

Name KEY
Date:
Section:

Each problem is worth 2 points. Total points: 32 points

Find the integral.

1) $\int (\sqrt{x} + \sqrt[3]{x})\, dx$ 1) C

A) $\frac{1}{2}x^{3/2} + \frac{2}{3}x^{4/3} + C$

B) $2\sqrt{x} + 2\sqrt[3]{x} + C$

C) $\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$

D) $2\sqrt{x} + 3\sqrt[3]{x} + C$

$\int (x^{1/2} + x^{1/3})\, d = \boxed{\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + c}$

2) $\int \frac{3\sqrt{x} - 5}{x^2}\, dx$ 2) B

A) $\frac{6}{\sqrt{x}} - \frac{5}{x} + C$

B) $-\frac{6}{\sqrt{x}} + \frac{5}{x} + C$

C) $-\frac{6}{\sqrt{x}} - \frac{5}{x} + C$

D) $\frac{6}{\sqrt{x}} + \frac{5}{x} + C$

$\int \left(\frac{3x^{1/2}}{x^2} - \frac{5}{x^2}\right) dx = \int (3x^{-3/2} - 5x^{-2})\, dx$

$= 3 \cdot \left(\frac{-2}{1}\right) x^{-\frac{1}{2}} - \frac{5x^{-1}}{-1} + c = \boxed{-6x^{-1/2} + \frac{5}{x} + c}$

3) $\int (t^4 + e^{4t})\, dt$ 3) A

A) $\frac{t^5}{5} + \frac{e^{4t}}{4} + C$

B) $\frac{t^3}{3} + 4e^{4t} + C$

C) $\frac{t^5}{5} + e^{4t} + C$

D) $\frac{t^5}{5} + \frac{e^{5t}}{5} + C$

$\int (t^4 + e^{4t})\, dt = \boxed{\frac{t^5}{5} + \frac{1}{4}e^{4t} + c}$

$u = 4t$

$du = 4dt$

4) $\int\left(\frac{1}{x}+\frac{2}{x^2}+\frac{3}{x^3}\right)dx$

4) C

A) $2x + 2\ln|x^2| + 3\ln|x^3| + C$

B) $\frac{2}{x^2}+\frac{6}{x^3}+\frac{12}{x^4}+C$

C) $\ln|x| - \frac{2}{x} - \frac{3}{2x^2} + C$

D) $\ln|x| + 2\ln|x^2| + 3\ln|x^3| + C$

$$\int\left(x^{-1}+2x^{-2}+3x^{-3}\right)dx = \ln|x| + \frac{2x^{-1}}{-1} + \frac{3x^{-2}}{-2} + c$$

$$= \boxed{\ln|x| - \frac{2}{x} - \frac{3}{2x^2} + c}$$

5) $\int\frac{\sqrt{x}-8}{2x\sqrt{x}}\,dx$

5) B

A) $\frac{1}{2}\sqrt{x}\ln|x| - 8x^{-1/2} + C$

B) $\frac{1}{2}\ln|x| + 8x^{-1/2} + C$

C) $\frac{1}{2}\sqrt{x}\ln|x| + 8x^{-1/2} + C$

D) $\frac{1}{2}\ln|x| - 8x^{-1/2} + C$

$$\int\left(\frac{x^{1/2}}{2x^{3/2}} - \frac{8}{2x^{3/2}}\right)dx = \int\left(\frac{1}{2}x^{-1} - 8x^{-3/2}\right)dx$$

$$= \frac{1}{2}\ln|x| - 8\cdot\left(\frac{-2}{1}\right)x^{-1/2} + c$$

$$= \boxed{\frac{1}{2}\ln|x| + 16x^{-1/2} + c}$$

6) $\int 9z\sqrt{3z^2-7}\,dz$

6) B

A) $\frac{1}{2}(3z^2-7)^{3/2} + C$

B) $(3z^2-7)^{3/2} + C$

C) $z(3z^2-7)^{3/2} + C$

D) $\frac{1}{2}z(3z^2-7)^{3/2} + C$

$u = 3z^2 - 7$

$du = 6z\,dz$

$$\frac{9}{6}\int(3z^2-7)^{1/2}\cdot 6z\,dz = \frac{3}{2}\cdot\frac{2}{3}(3z^2-7)^{3/2} + c$$

$$= \boxed{(3z^2-7)^{3/2} + c}$$

7) $\int \frac{x}{(7x^2+3)^5}\,dx$

7) C

A) $\frac{-7}{3(7x^2+3)^6} + C$

B) $\frac{-1}{14(7x^2+3)^6} + C$

C) $\frac{-1}{56(7x^2+3)^4} + C$

D) $\frac{-7}{3(7x^2+3)^4} + C$

$\int x\,(7x^2+3)^{-5}\,dx = \frac{1}{14}\int (7x^2+3)^{-5}\,14x\,dx = \frac{1}{14}\,\frac{(7x^2+3)^{-4}}{-4} + c$

$u = 7x^2+3$

$du = 14x\,dx$

$= \boxed{-\frac{1}{56(7x^2+3)^4} + c}$

8) $\int (1-6x)e^{3x-9x^2}\,dx = \frac{1}{3}\int e^{3x-9x^2}\,(3)(1-6x)\,dx$

8) $\frac{1}{3}e^{3x-9x^2} + c$

$u = 3x-9x^2$

$du = (3-18x)\,dx$

$= 3(1-6x)\,dx$

$= \boxed{\frac{1}{3}e^{3x-9x^2} + c}$

9) $\int \frac{t^2+1}{t^3+3t+4}\,dt = \frac{1}{3}\int \frac{1}{(t^3+3t+4)'}\cdot 3(t^2+1)\,dt$

9) $\boxed{\frac{1}{3}\ln|t^3+3t+4| + c}$

$u = t^3+3t+4$

ln

$du = (3t^2+3)\,dt$

$= 3(t^2+1)\,dt$

$= \boxed{\frac{1}{3}\ln|t^2+3t+4| + c}$

10) $\int \frac{1}{x(\ln x^3)}\,dx = \frac{1}{3}\int \frac{1}{(\ln x^3)'}\cdot\frac{3}{x}\,dx$

10) $\frac{1}{3}\ln|\ln x^3| + c$

$u = \ln x^3$

ln

$du = \frac{1}{x^3}\cdot 3x^2\,dx$

$= \boxed{\frac{1}{3}\ln|\ln x^3| + c}$

$du = \frac{3}{x}\,dx$

Evaluate.

11) $\int_1^e \left(8x - \frac{13}{x}\right) dx$

11) $4e^2 - 17$ ≈ 12.6

$= \frac{8x^2}{2} - 13\ln|x| \Big|_{x=1}^{x=e} = 4x^2 - 13\ln|x| \Big|_{x=1}^{x=e}$

$= \left(4(e)^2 - 13\underbrace{\ln|e|}_{1}\right) - \left(4(1)^2 - 13\underbrace{\ln|1|}_{0}\right)$

$= 4e^2 - 13 - 4 = \boxed{4e^2 - 13}$

12) $\int_1^3 \frac{x^4 - x^{-1}}{x^2} dx = \int_1^3 \left(\frac{x^4}{x^2} - \frac{x^{-1}}{x^2}\right) dx$

12) $\frac{74}{9} \approx 8.22$

$= \int_1^3 (x^2 - x^{-3}) dx = \frac{x^3}{3} - \frac{x^{-2}}{-2} \Big|_{x=1}^{x=3}$

$= \frac{1}{3}x^3 + \frac{1}{2x^2} \Big|_{x=1}^{x=3} = \left(\frac{1}{3}(3)^3 + \frac{1}{2(3)^2}\right) - \left(\frac{1}{3}(1)^3 + \frac{1}{2(1)^2}\right) =$

$= 9 + \frac{1}{18} - \frac{1}{3} - \frac{1}{2} = \frac{162 + 1 - 6 - 9}{18} = \frac{148}{18} = \frac{74}{9}$

13) $\int_0^6 \sqrt{6x}\, dx$

13) 24

$\int_0^6 \sqrt{6}\sqrt{x}\, dx = \sqrt{6} \int_0^6 x^{1/2} dx$

$= \sqrt{6}\left(\frac{2}{3}\right) x^{3/2} \Big|_{x=0}^{x=6} = \frac{2\sqrt{6}}{3} \left(x^{3/2} \Big|_{x=0}^{x=6}\right)$

$= \frac{2\sqrt{6}}{3}\left(6^{3/2} - 0^{3/2}\right) = \frac{2\sqrt{6}}{3}\left(6\sqrt{6} - 0\right)$

$= \frac{2\sqrt{6}}{3}\left(6\sqrt{6}\right) = \frac{2(\cancel{6}^{2})(6)}{\cancel{3}} = \boxed{24}$

Evaluate the integral.

14) $\int \frac{\sin t}{(8 + \cos t)^5} dt$ $\quad u = 8+\cos t$, $du = -\sin t\,dt$

14) A

A) $\frac{1}{4(8 + \cos t)^4} + C$

B) $\frac{1}{6(8 + \cos t)^6} + C$

C) $\frac{1}{(8 + \cos t)^4} + C$

D) $\frac{4}{(8 + \cos t)^4} + C$

$-\int (8+\cos t)^{-5} (-\sin t\,dt) = -\frac{(8+\cos 5t)^{-4}}{-4} + c$

$= \boxed{\frac{1}{4(8+\cos 5t)^4} + C}$

15) $\int \csc\left(z + \frac{\pi}{6}\right) \cot\left(z + \frac{\pi}{6}\right) dz$ $\quad u = z+\frac{\pi}{6}$, $du = dz$

15) B

A) $-\frac{\pi}{6} \csc\left(z + \frac{\pi}{6}\right) + C$

B) $-\csc\left(z + \frac{\pi}{6}\right) + C$

C) $-\cot\left(z + \frac{\pi}{6}\right) + C$

D) $\csc\left(z + \frac{\pi}{6}\right) + C$

Recognize that

$\frac{d}{dx}(\csc x) = -\csc(x)\cot(x)$

$= \boxed{-\csc\left(z+\frac{\pi}{6}\right) + c}$

16) $\int \sin (2x - 5)\, dx$ $\quad u = 2x-5$, $du = 2dx$

16) C

A) $\frac{1}{2} \cos (2x - 5) + C$

B) $-\cos (2x - 5) + C$

C) $-\frac{1}{2} \cos (2x - 5) + C$

D) $2 \cos (2x - 5) + C$

$= \frac{1}{2}\int \sin(2x-5) \cdot 2dx = \boxed{-\frac{1}{2}\cos(2x-5) + c}$